Three-Dimensional Nonlinear Finite Element Analysis of the Macroscopic Compressive Failure of Concrete Materials Based on Real Digital Image

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Abstract

In this paper, a three-dimensional (3-D) finite element (FE) analysis procedure for the macroscopic compressive failure of concrete materials is described and a numerical example is shown. A 3-D digital image of concrete materials is directly used for geometrically accurate FE modeling with digital image processing. Concrete materials are modeled as two-phase composites consisting of coarse aggregates, mortar, and the interfaces between them to explain the macroscopic compressive failure from local tensile fractures.

Introduction

In many practical stress analyses of concrete structures, concrete materials are regarded as homogenous materials and constitutive laws to represent their nonlinear stress-strain relations are phenomelogical models. This is enough to predict the mechanical behaviors of concrete structures. However, from a microscopic point of view, concrete materials are multi-phase composite materials. It has been recognized that local tensile failures derived from the material heterogeneities are the most essential mechanical behaviors. As a consequence of their accumulation, mechanical behaviors of concrete materials can be observed as nonlinear stress-strain relations from a macroscopic point of view. The relation between the macroscopic behaviors and the local tensile failures is complex and it has not been almost clear in detail, especially for compression.

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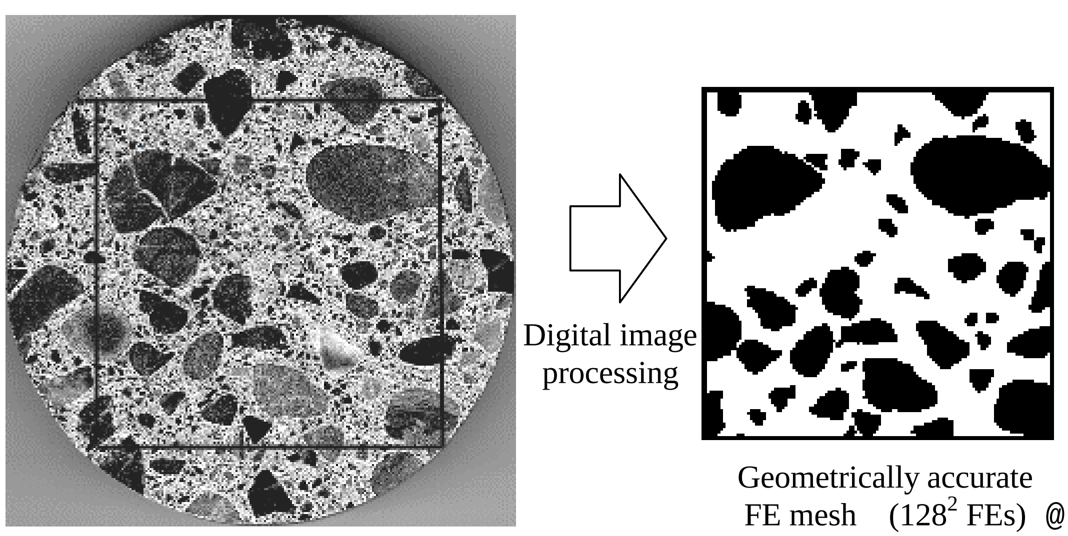
The objective of this work is to explain the macroscopic mechanical behaviors of concrete materials subjected to compression from the local tensile failures by numerical analysis. If they can be explained, macroscopic mechanical behaviors of concrete materials subjected to tension can be also explained. Such a numerical analysis of concrete materials may give much useful information to the more precise modeling of the constitutive laws. However, numerical analysis of concrete materials seem to be not yet enough to do it. Therefore, this paper proposes a new FE analysis procedure for the compressive failure of concrete materials by using a real digital image.

Finite Element Modeling Based on Real Digital Image of Concrete Materials

At the first step of the numerical analysis of concrete materials, proper geometric modeling as composites is required to capture local tensile failures in concrete materials caused by macroscopic compression. A two-phase system consisting of coarse aggregates and homogeneous mortar should be modeled due to the size effect of fracture mechanics, although various smaller constituents exist in mortar. This corresponds to the experimental result of Cordon and Gillespie (1963) that the maximum size of coarse aggregates considerably influences the compressive strength. It is clear from this result that various geometric properties of coarse aggregates such as their size, shape and configuration influence the mechanical behaviors.

Thus, the proper geometric modeling is important. However, it is difficult because the geometry of the coarse-aggregates-and-mortar system is complex in 3-D space. Moreover, with the conventional FE method, FE meshing is very difficult



A section of concrete specimen

Fig. 1 2-D digital image-based FE modeling because the mesh have to be fitted to the 3-D complex geometry.

To avoid these difficulties, we directly use a real digital image acquired from a physical concrete specimen (Nagai et al. 1998, 2000a). The original concept of the use of digital images has been proposed by Hollister and Kikuchi (1994) in the field of biomechanics. In the 2-D case, a digital image consists of a small square-shape element whose center is a sampling point. The small square element is called a *pixel* (picture cell). If one pixel is regarded as one 4-node square-shape FE, a domain can be divided into the same shape FEs. Material properties of individual FE can be determined by thresholding the gray level on the sampling point. That is, the entire FE meshing procedures can be replaced by digital image processing as shown in Fig. 1. Obviously, the digital image-based FE modeling is geometrically accurate if appropriate digital images can be obtained.

This digital image-based FE modeling can be easily extended to the 3-D case. A 3-D digital image is a stack of 2-D sectional digital images. As well as the 2-D case, one small element called a *voxel* (volume cell) can be regarded as one 8-node cubic-shape FE. The sequence of the sectional digital images is acquired by the repetition of scraping a physical concrete specimen and scanning the section as shown in Fig. 2-a. After image processing, a coarse-aggregates-and-mortar model shown in Fig. 2-b is obtained. This model has 320^3 voxels and the number of degrees of freedom is about 100 millions. We have developed a fast and efficient linear equation solver using the features of a digital image and the homogenization method (Guedes and Kikuchi 1990), and carried out the linear stress analysis of this model (Nagai et al. 2000b).

However, in exchange for the geometrically accurate FE modeling, a new

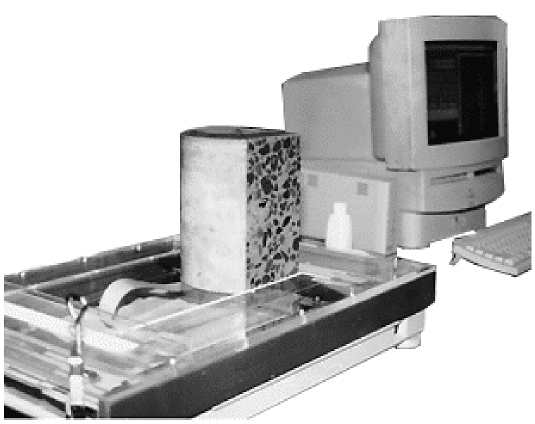


Fig. 2-a Acquiring a sectional digital image of concrete specimen by scanner

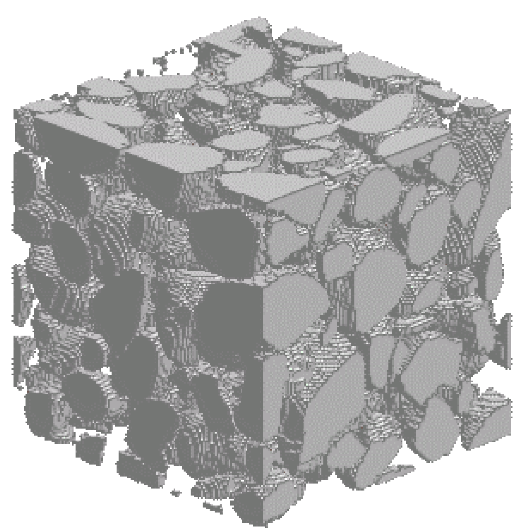


Fig.2-b Configuration of coarse aggregates in 3-D digital image of concrete materials

problem arises from a feature of digital images. That is, the interfaces between coarse aggregates and mortar are approximated to jagged. In addition, it has been recognized that local tensile failures observed as cracks initially occur along the interfaces, and launch into coarse aggregates or mortar. Therefore, the following section describes an improvement of the digital image-based FE modeling to represent the interfaces and the interfacial cracks more precisely.

High-Resolution Modeling of Coarse-Aggregates-and-Mortar Interfaces

We now define the interfaces between coarse aggregates and mortar as shown in Fig. 3. The interfaces are independent of the digital FE mesh. The FEs

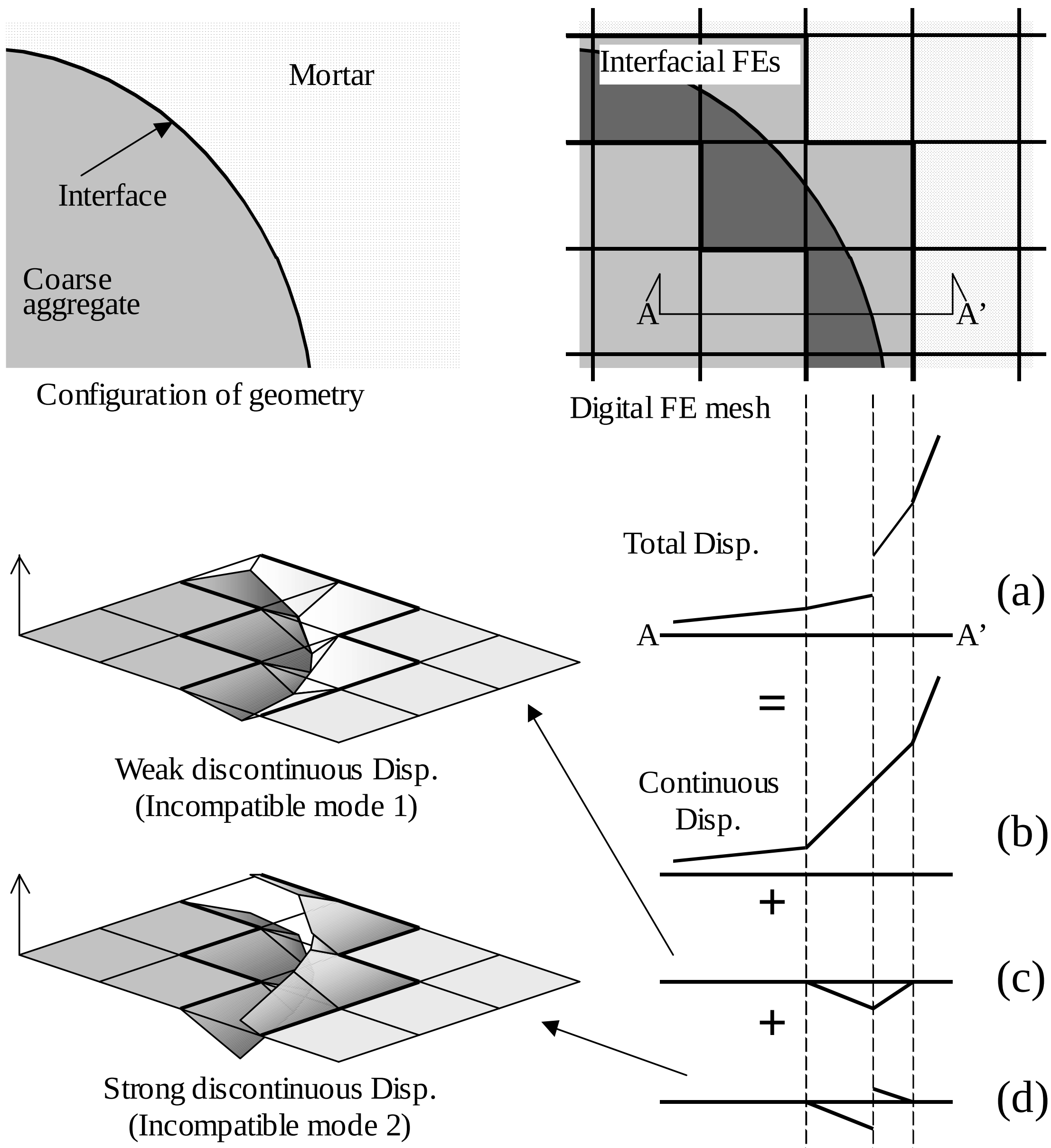


Fig. 3 Definition of displacement functions on 2–D digital FE mesh

containing the interfaces are referred to *interfacial FEs*. We propose a new mixed formulation for the interfacial FEs (Nagai 2000a), which is based on the assumed enhanced strain (AES) method proposed by Simo and Rifai (1990). To make the FE mesh from a 3-D digital image of concrete materials, a 3-D edge-based contouring algorithm can be employed. This algorithm is known as *iso-surface* of discretized volume data in the field of medical image or scientific visualization.

A mixed formulation based on the assumed enhanced strain method

The displacement field in the vicinity of the interfacial FEs can be modeled as (a) in Fig. 3. According to the approach of Oliver (1996a, 1996b), the displacement function (a) can be decomposed into three functions (c)-(d). (b) is a continuous function, (c) is a weak discontinuous function related to the strain jump derived from the presence of two different materials, and (d) is a strong discontinuous function related to the displacement jump derived from the interfacial crack. In the AES method, (a) is taken into the mixed formulation as compatible mode, and (c) and (d) as incompatible modes. The incompatible modes are appeared only in the interfacial FEs. The rests are ordinary FEs as described in the previous section. For individual interfacial FE, traction continuity on an interface by means of integration is imposed as the constraint condition of the AES method. This ensures the satisfaction of the patch test (Simo and Rifai 1990).

Finite element meshing with 3-D edge-based contouring

We now adopt another definition of a voxel to regard as an 8-node cubic-shape FE. A voxel is defined in eight sampling points that forms a cube. Each sampling point can be classified into coarse aggregates or mortar region by thresholding the gray level at a certain level θ. Let us consider an edge of the cube. If both sampling points connecting to the edge do not belong to the same region, an intersectional point of the edge and the interface exists. The intersectional point $\boldsymbol{x}$ can be computed from the gray level f on the two sampling points $\boldsymbol{x}_1$, $\boldsymbol{x}_2$ with the following linear interpolation;

$$\mathbf{x} = \frac{\left(f(\mathbf{v}_1) - \theta\right)\mathbf{v}_2 + \left(\theta - f(\mathbf{v}_2)\right)\mathbf{v}_1}{f(\mathbf{v}_1) - f(\mathbf{v}_2)}.$$

With the algorithm of Nagae et al. (1993), all intersectional points in an interfacial FE

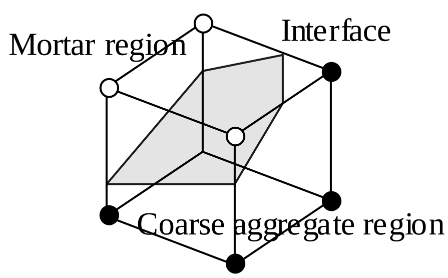


Fig. 4-a Interface in an interfacial FE

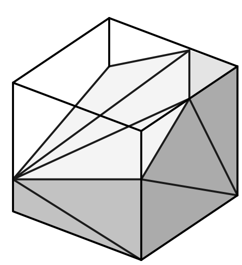


Fig.4-b Subdivision into tetrahedrons

can be connected systematically. For example, Fig. 4-a shows an interface in an interfacial FE. Integration in the interfacial FE can be evaluated by subdividing its whole domain into tetrahedrons as shown in Fig. 4-b. It is noted that if no intersectional point exists in a voxel, the voxel is an ordinary FE.

A Numerical Example

Constitutive law for coarse-aggregates-and-mortar interfaces

As a constitutive law for the interfaces between coarse aggregates and mortar, the isotropic continuum damage model of Oliver (1996a) is used. In this model, only tensile failures are taken into account. The tensile failures occur if at least one of stresses in the principal stress coordinate system shown in Fig. 5 exceeds a certain tensile strength limit. Tensile softening monotonically follows the exponential curve with consideration of the fracture energy, and unloading goes toward the origin. Initial linear elastic modulus is used in pure compression domain. The material properties used in this example are shown in Table 1.

Uni-axial macroscopic compression

A real 3-D digital image of concrete materials shown in Fig. 6-a is used. This model has 48^3 FEs. Interfaces in the digital FE mesh are shown in Fig. 6-b. The macroscopic structure is subjected to vertical compression by using the homogenization method (Guedes and Kikuchi 1990). The nonlinear equation is solved by using the secant stiffness method. The macroscopic stress-strain relation is shown in Fig. 6-c. The distribution of minimum principal stress at the point A in Fig.6-c is shown in Fig. 6-d.

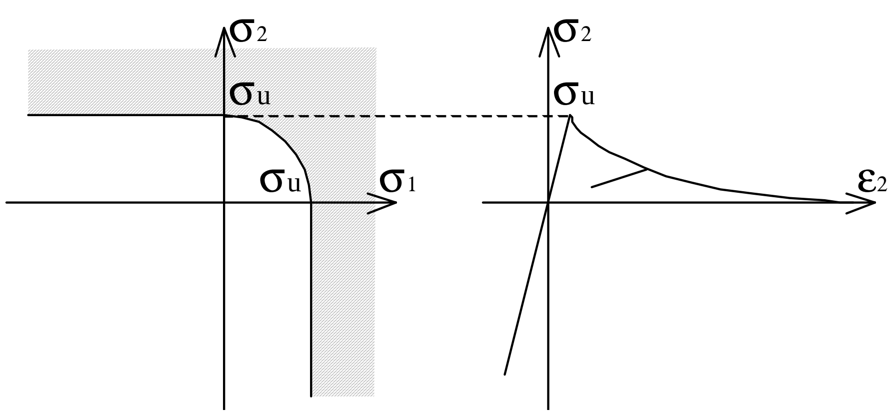


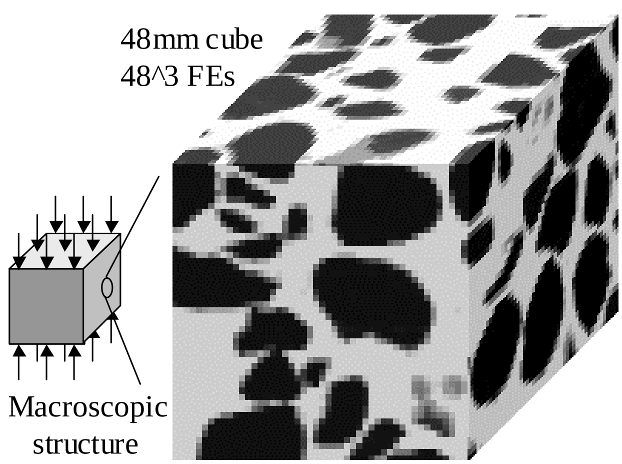
Fig. 5 Criterion of damage progression and constitutive law of interface

Table 1 Material properties

	Young's modulus	Poisson's ratio	Fracture energy	Tensile strength
Coarse Agg.	69 GPa	0.20	-	-
Mortar	25 GPa	0.20	-	-
Interface	250 GPa/mm	0.20	4.9 J/m^2	1.2 MPa

Concluding Remarks

Digital image-based FE modeling of concrete materials consisting of coarse aggregates and mortar has been described. This FE modeling is geometrically accurate in spite of its 3-D complex geometry. The interfaces between coarse aggregates and mortar can be modeled by using a new mixed FE and a 3-D edge-based contouring algorithm. The first stage of the macroscopic compressive mechanical behavior of concrete materials can be exhibited by modeling only local tensile failures on the interfaces. If the local tensile failures launching into coarse aggregates or mortar are taken into account, we could obtain the point of macroscopic compressive strength.



Coarse aggregates and mortar

Fig. 6-a Macroscopic compression test

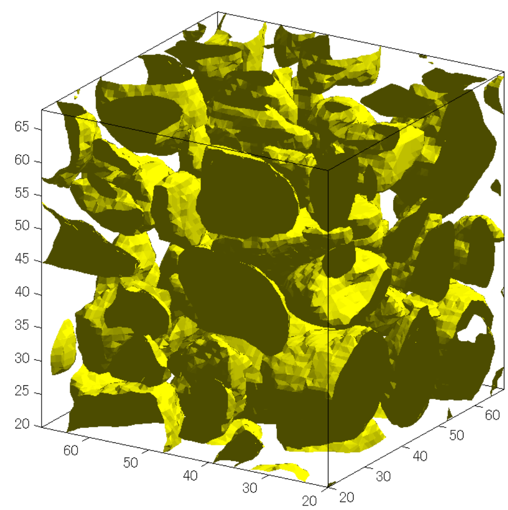


Fig. 6-b Interfaces

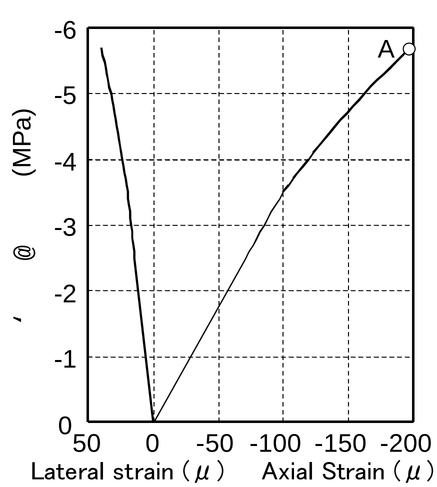


Fig. 6-c Macroscopic stress-strain relation

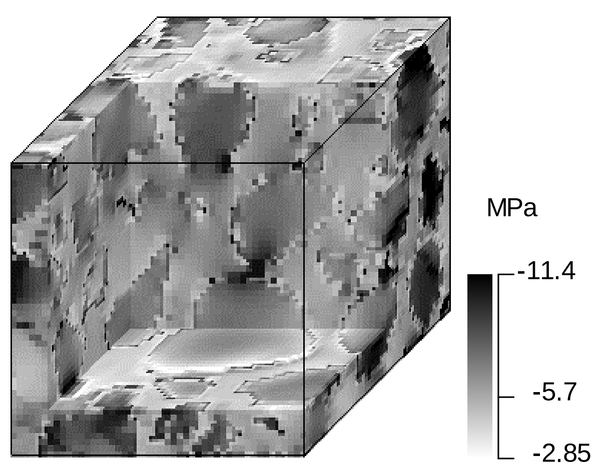


Fig. 6-d Minimum principal stress at point A

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