# VIBRATION CONTROL DEVICES FOR TENSION STRUCTURES USING VISCO-ELASTIC OR VISCOUS MATERIAL WITH SPRING

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## ABSTRACT

Additional damping mechanisms using Visco-(elastic) materials with springs for tension structures are proposed. Their characteristics are modeled by Voigt model and expressed by simple equations using stiffness ratios of each part, followed by verification by real-size mock-up tests.

## 1.INTRODUCTION

Tension strings in tension structures are widely applicable as additional stiffness / strength elements in space structures. However, compared with building frames based on the elast-plastic design, these members are basically designed elastically and have no energy absorption mechanisms against wind and earthquake forces. If additional damping mechanisms for general tension strings are available, they will contribute greatly to the expansion of the application scope of tension structures. In detail, they are expected to stabilize and reduce the response against seismic vibration situated in the short-natural frequency, or to prevent the aerodynamically unstable vibration of lightweight and low-stiffness roofs subjected to wind forces.

Attempts to impart additional damping to tension structures have been sporadically made in the past[1], however, no studies are found that evaluated the performance of additional dampers installed in series with strings as parts of a general-purpose structural system and proposed generalized design methods

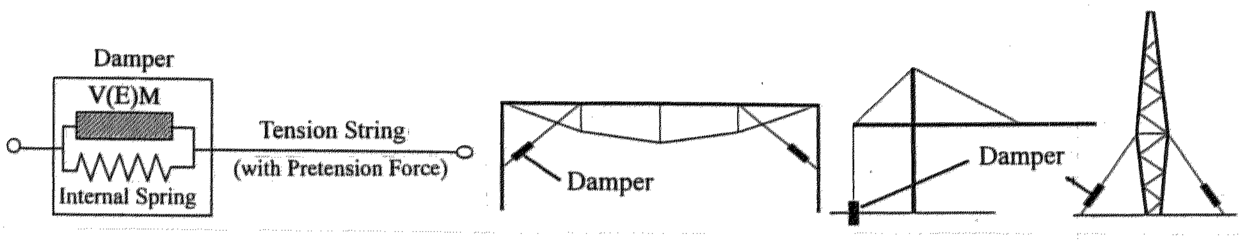



Fig.1. V(E)M with Spring as Tension-String Damper

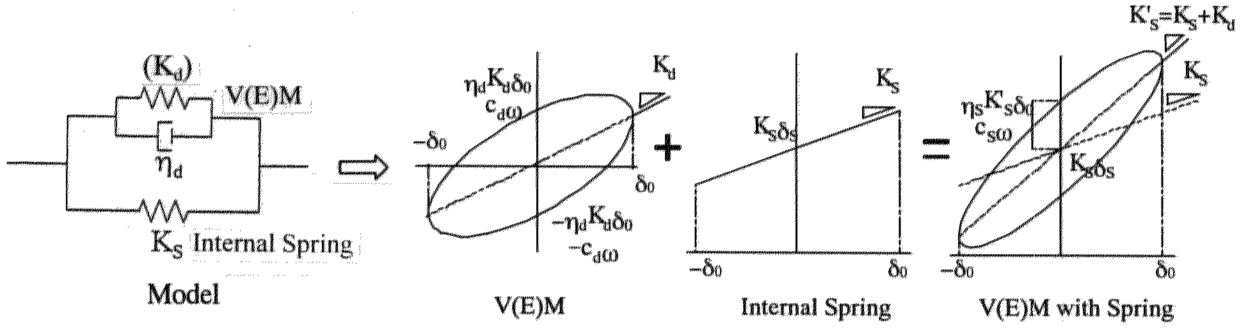



Fig.2. V(E)M with Spring by Voigt Model

for additional dampers.

As the dampers for tension structures address not only earthquake forces but also wind forces, viscous materials (VM) or visco-elastic materials (VEM) having high-cycle repetitive damping performance are considered suitable. VM or VEM do not have displacement-proportional springs by themselves, so that when these materials are arranged in series with strings, they cannot retain the pre-tension forces introduced into the strings. However, when a V(E)M is combined with an elastic spring as shown in Fig. 1, an additional damper can be constructed for a tension string with pretension force. In this case, V(E)M produces a damping effect on the elastic displacement of the internal spring. Spring-loaded V(E)M connected to tension strings can be used to various tension structures as shown in Fig. 1.

Behavior of V(E)M have been extensively investigated by methods using fractional derivatives or methods using multiple-element models and applied as analytical techniques for random waves changing in frequency[2)]. However, for the space structures whose vibration mode is predominantly governed by first-mode, there are expected to exist a range where the simple Voigt model can be directly applied.

## 2. DERIVATION OF GENERALIZED EQUATIONS

When a V(E)M is represented by a Voigt model, the hysteresis loop of a spring-loaded V(E)M can be represented by the addition of the combined elastic stiffness of V(E)M and the internal spring to the viscous hysteresis of the V(E)M, as shown in Fig. 2. For VM can be modeled by special case of $K_d$=0, we start with VEM firstly. VEM usually has large temperature and frequency dependence, and the reference 3) gives the shearing stiffness ratio $k_d$ and the damping factor c by the following simple equations for acrylic VEM.

$$k_d = a_0 f^{a1} \gamma_a^{a2} e^{a3q} \tag{1}$$

$$c = b_0 f^{b1} \gamma_a^{b2} e^{b3q} \tag{2}$$

where f is the frequency (Hz); $\gamma_a$ is the shear strain: q is the temperature (°C); and $a_0$, $a_1$, $a_2$, $a_3$, $b_0$, $b_1$, $b_2$, and $b_3$ are constants set for specific materials. Shear stiffness ratio $K_d$, and loss factor $\eta = c\omega/K_d$ of this visco-elastic material are then given by;

$$K_d = k_d \frac{A_s}{t} \qquad \eta = c\omega / K_d \tag{3}$$

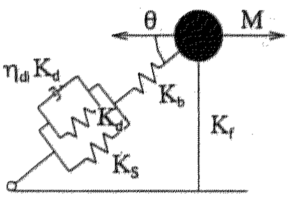



Fig.3. Structure with Tension Damper

where $A_S$ is the total area of VEM ($cm^2$): and t is the thickness of VEM (cm). The load-deformation relationship at constant amplitude and frequency is given by;

$$Q_d(t) = K_d\delta(t) \pm \eta_d K_d\sqrt{\delta_0^{\,2} - \delta(t)^2} \tag{4}$$

Let $\delta_S$ denote the initial deformation by the pretension force and $K_S$ (kN/cm) denote the internal spring constant. The load-deformation relationship of the VEM with the spring is then given by;

$$\begin{aligned} Q(t) &= Q_d(t) + (\delta(t) + \delta_S)K_S \\ &= (K_d + K_S)\delta(t) \pm \eta_d K_d\sqrt{\delta_0^{\,2} - \delta(t)^2} + \delta_S K_S \end{aligned} \tag{5}$$

The equivalent stiffness and load factor can be thus derived as follows:

$$K'_S = K_d + K_S \tag{6}$$

$$\eta_S = \frac{K_d}{K_d + K_S}\eta_d \tag{7}$$

Let us now consider the structures in which the additional damping elements are arranged in series with strings. These structures can be all simplified to the generalized model of Fig. 3 by representing their vibration by first-mode vibration. Let $K_b$ (kN/cm) denote the axial stiffness of the string, $P = \delta_S K_S$ denote the pretension force of the string. From the displacement-load relationship of the model in which the damper is combined in series with the string.,

$$Qa(t) = K_a\delta_a(t) \pm \eta_a K_a\sqrt{\delta_{a0}^{\,2} - \delta_a(t)^2} + P \tag{8}$$

where,

$$K_a = \frac{1}{\dfrac{1}{K'_S\Gamma_S} + \dfrac{1}{K_b}} \qquad \Gamma_S = 1 + \frac{\eta_S^{\,2}}{1 + \dfrac{K'_S}{K_b}} \tag{9}$$

$$\eta a = \frac{\eta_S}{1 + (1 + \eta_S^{\,2})\dfrac{K'_S}{K_b}} \qquad \delta_{a0} = \delta_0\sqrt{\eta_S^{\,2}\left(\frac{K'_S}{K_b}\right)^2 + \left(1 + \frac{K'_S}{K_b}\right)^2} \tag{10}$$

The load-deformation relationship of the frame to which the string with the damper is attached at the angle θ is similarly given by the following if the stiffness of the frame is denoted by $K_f$ (kN/cm) and $\delta_f(t) = \delta_a(t)/\cos\theta$:

$$Q_{eq}(t) = K_{eq}\delta_f(t) \pm \eta_{eq}K_{eq}\sqrt{\delta_{eq^2} - \delta_f(t)^2} + P\cos\theta \tag{11}$$

where,

$$K_{eq} = K_f + K_a\cos\theta \qquad \eta_{eq} = \frac{\eta_a}{1 + \dfrac{K_f}{K_a\cos^2\theta}} \qquad \delta_{eq} = \delta_{a0}/\cos\theta \tag{12}$$

From the above equations, the equivalent damping coefficient of the overall structural system $h_{eq}=\eta_{eq}/2$ and equivalent period $T_{eq}=2\pi\sqrt{M/K_{eq}g}$ (where M is the mass of the frame (ton)) are obtained. When $K_b = \infty$, Eq. (12) becomes:

$$h_{eq} = \frac{\eta_d}{1 + \dfrac{K_S}{K_d} + \dfrac{K_f}{K_d\cos^2\theta}} \tag{13}$$

This suggests that when the frame stiffness ratio $K_f/K_d$ is constant, the equivalent damping of the entire structural system increases with decreasing stiffness ratio $K_S/K_d$ of the internal spring.

In the same process, in case that viscos material (VM) are used instead of VEM, eq.(9)-(10) are replaced by the following, and eq.(11)-(12) can be directly applied with them.

$$K_a = \frac{1}{\dfrac{1}{K_S + \dfrac{c_d{}^2\omega^2}{K_S + K_b}} + \dfrac{1}{K_b}} \tag{14}$$

$$\eta_a K_a = c_a\omega = \frac{c_d\omega}{\left(1 + \dfrac{K_S}{K_b}\right)^2 + \left(\dfrac{c_d\omega}{K_b}\right)^2} \tag{15}$$

$$\delta_{a0} = \delta_0\sqrt{\left(1 + \frac{K_S}{K_b}\right)^2 + \left(\frac{c_d\omega}{K_b}\right)^2} \tag{16}$$

where, $c_d$:damping factor of VM, $\omega$:circular frequency.

## 3.ESTIMATIONS OF EQUIVALENT DAMPING

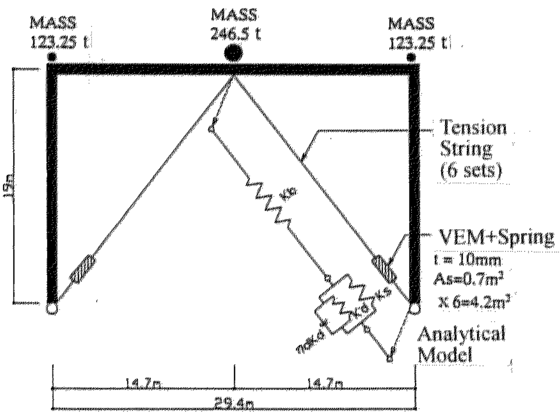
A moment frame stiffened by tension strings as shown in Fig. 4 are set as the model to verify the validity of Eqs. (3) to (16) derived. Here uses the horizontal forces as disturbance for discussion. The internal spring stiffness ratio, connected string stiffness ratio and frame stiffness ratio with respect to the V(E)M, and the ambient temperature are changed,and equivalent additional damping $h_{eq}$ and equivalent frequency $f_{eq}$ of the entire structure are calculated and summarized in Table 1.

The stiffness of the moment frame without dampers and strings are set so that the natural frequency becomes 2.0 Hz. The connected string stiffness is determined to cause $K_b/K_{d0}$ to become 2, 4, or 20 with respect to the VEM stiffness $K_{d0}$, and $K_b/c_d$ to become 60, 120, 600 with respect to the VM damping $c_d$. These values correspond to steel bars with diameters of 64, 90, and 200 mm, respectively, in the setup model. The internal spring stiffness is set in four cases of $K_s/K_{d0}$ = 0, 1, 2, 4 in VEM, and.$K_s/c_d$ =0, 30, 60,120 in VM. In the case of $K_s/K_{d0}$ = 1 of VEM, the ambient temperature is set at three levels of 10, 20, and 30°C.

An FEM model of the moment frame was built for VEM cases for the purpose of verification. Using the fractional derivative elements (Kasai[2)]) proven for performance in analysis of VEM, time history response analysis with respect to step loads was conducted, and $h_{eq}$ and $f_{eq}$ were calculated from the response histories. The values given in Table 1 are the theoretical values obtained by Eqs. (3) to (16), and the values enclosed with parentheses are the analytical values determined by the derivative element method. The constants in Eqs. (1) and (2) and the derivatives use the values shown in Fig.4.

Of the above-mentioned results, time-displacement relationship and hysteresis loop against step load are shown in Fig.5. Also the relationship between the equivalent damping constant and the internal spring stiffness ratio at the temperature of 20°C is shown in Fig. 6a). The tendency for the equivalent damping constant to increase with decreasing spring stiffness ratio is consistently confirmed, not only when $K_b = \infty$, but also when the string stiffness ratio $K_b/K_{d0}$ is reduced. This means that the internal spring can be designed with the lowest possible stiffness required for the control of the VEM.

Figure 6b) shows the relationship between the equivalent damping constant and the ambient temperature when $K_s/K_{d0} = 1$ in VEM. When the connected string stiffness is high ($K_b/K_{d0} = 20$), the equivalent damping constant sharply increases as the ambient temperature lowers. This tendency directly reflects the temperature dependence of the VEM itself. However, when $K_b/K_{d0}$ drops to 4 or less, the




Analysis Model

Parameters for VEM

| Parameters in Kekvin-Voigt Model | | | | |
|---|---|---|---|---|
| $K_d$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ |
| | 8.57 | 0.30 | -0.24 | -0.07 |
| $c$ | $b_0$ | $b_1$ | $b_2$ | $b_3$ |
| | 2.18 | -0.53 | -0.09 | -0.10 |
| **Parameters in Kasai Model** | | | | |
| $a_{ref}$ | $b_{ref}$ | $\alpha$ | G(kgf/cm²) | $Th_{ref}$(°C) |
| 0.0115 | 21.16 | 0.6089 | 0.333 | 0.2 |

Fig.4. Structural Model for Time-Analysis

Table1. Equivalent Damping and Frequency

$K_{d0}$: VEM Stiffness at 20C, 250% (kN/cm),
$C_d$: VM Damping (kNsec/cm) Ks: Spring Stiffness (kN/cm),
$K_b$: Tension Member Stiffness (kN/cm), $K_f$: Frame Stiffness (kN/cm)
$f_0$: Frequency for Frame only (Hz),
heq: Total Equivarent Damping Ratio (%),
$f_{eq}$: Total Equivarent Frequency (Hz)

| VEM | $K_f/K_{d0}$ | 1 ($K_f$=800kN/cm, $K_{d0}$=840kN/cm) | | | | | |
|---|---|---|---|---|---|---|---|
| | $K_b/K_{d0}$ | 2 (Kb=1680) | | 4 (Kb=3360) | | 20 (Kb=16800) | |
| $K_s/K_{d0}$ | Temp. | heq | $f_{eq}$ | heq | $f_{eq}$ | heq | $f_{eq}$ |
| 0 (Ks=0) | 20 | 9.8 (13.3) | 2.7 | 15.6 (19.6) | 2.8 | 25.2 (29.7) | 2.9 |
| 1 (Ks=840) | 10 | 5.7 (6.0) | 3.0 | 11.4 (11.8) | 3.5 | 30.1 (33.8) | 4.1 |
| | 20 | 5.8 (7.4) | 2.8 | 9.9 (12.7) | 3.0 | 18.3 (23.2) | 3.4 |
| | 30 | 3.6 (5.1) | 2.6 | 5.6 (7.8) | 2.8 | 8.6 (12.4) | 3.0 |
| 2 (Ks=1680) | 20 | 3.8 (4.7) | 2.9 | 6.9 (8.7) | 3.2 | 14.3 (18.4) | 3.7 |
| 4 (Ks=3360) | 20 | 2.0 (2.4) | 3.0 | 4.0 (4.9) | 3.4 | 9.8 (12.3) | 4.3 |

| VM | $K_f/c_d$ | 28($K_f$=800kN/cm, $c_d$=28kNsec/cm) | | | | | |
|---|---|---|---|---|---|---|---|
| | $K_b/c_d$ | 60 ($K_b$=1680) | | 120 ($K_b$=3360) | | 600 ($K_b$=16800) | |
| $K_S/c_d$ | *$K_S/c_d\omega$ | $h_{eq}$ | $f_{eq}$ | $h_{eq}$ | $f_{eq}$ | $h_{eq}$ | $f_{eq}$ |
| 0 ($K_S$=0) | 0 | 15.2 | 2.08 | 16.1 | 2.04 | 16.5 | 2.01 |
| 30 ($K_S$=840) | 1.9 | 5.7 | 2.5 | 8.1 | 2.6 | 11.3 | 2.66 |
| 60 ($K_S$=1680) | 3.5 | 3 | 2.7 | 5.1 | 2.89 | 8.7 | 3.14 |
| 120 ($K_S$=3360) | 6.4 | 1.3 | 2.88 | 2.6 | 3.23 | 6 | 3.82 |

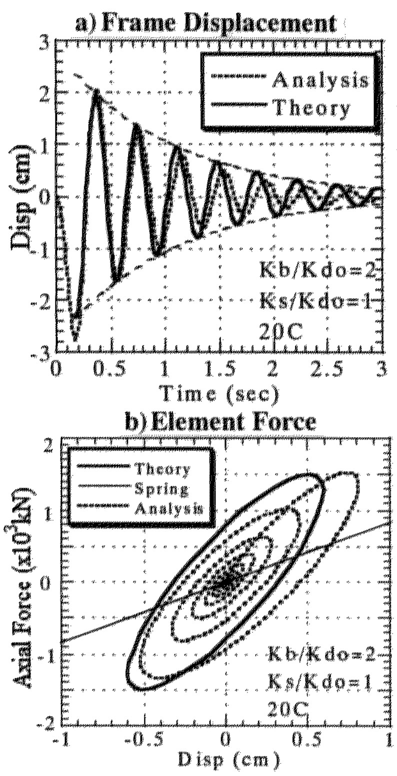



Fig.5. Results against Step Load

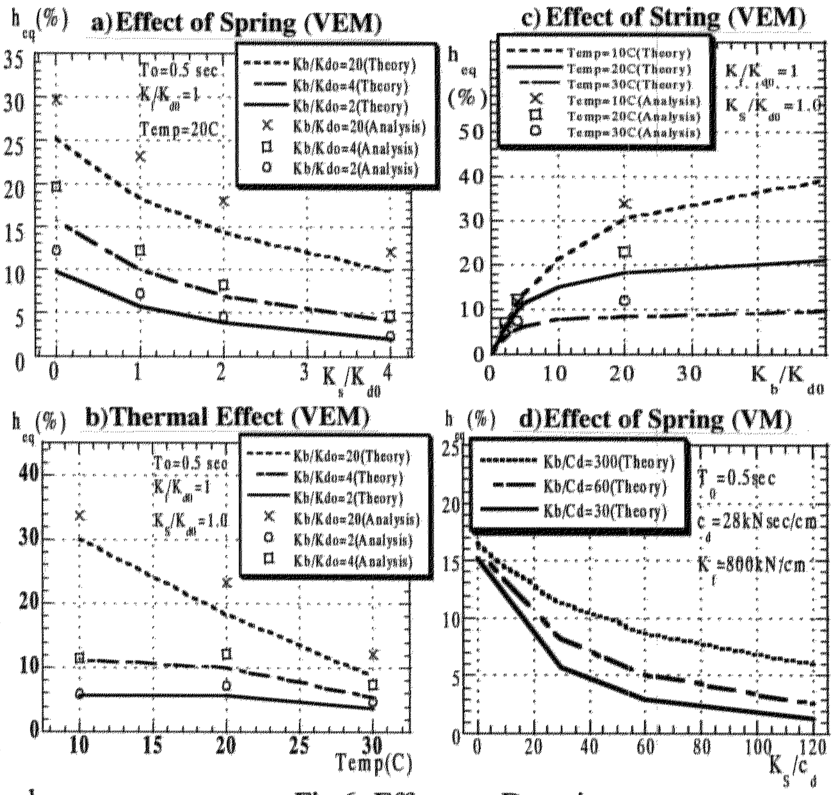



Fig.6. Effects on Damping

equivalent damping constant declines on the whole with decreasing tension string stiffness ratio, and the change with the ambient temperature becomes practically flat. This is attributable to the increased effect of the second term in denominator in Eq. (9). Figure 6c) shows the relationship between the equivalent damping constant and the connected string stiffness ratio $K_b/K_{d0}$. The equivalent damping constant is different for different temperatures in the $K_b/K_{d0}$ range of 30 and above for usual VEM braces and is low in difference with the temperature in the $K_b/K_{d0}$ range of about 1 to 10 for tension strings. The relaxation phenomenon of this temperature dependence can be considered characteristic of tension structures of low connected string stiffness ratio. The above-mentioned characteristics are supported by the analytical results of the FEM model obtained by using the derivative elements and shown as the analytical values in Figs. 6.

Equivalent damping for VM with spring are also shown in Fig. 6d), which shows the similar cgaracteristics as VEM, however, effects of internal spring are more remarkable than VEM.

## 4.REAL-SIZE MOCK-UP TEST

To verify the analytical results, real size mock-ups of tension damper using VEM are composed and dynamic loading tests are conducted. Details of mock-up specimen and test set-up are shown in Fig.7. Mock-up specimens are composed of double layers of steel pipes, with VEM fixed in-between. Steel springs are also inserted between these two pipes, VEM and springs working together against deformation between two pipes by one pipe is connected to tension strings and other is connected to dynamic actuator. VEMs' thickness are 8mm, areas are $1038cm^2$ or 519cm. The stiffness of internal springs are $K_S$=1.32kN or 2.73kN. Tension strings in line are varied in three types of $K_b=\infty$, 12.17N/

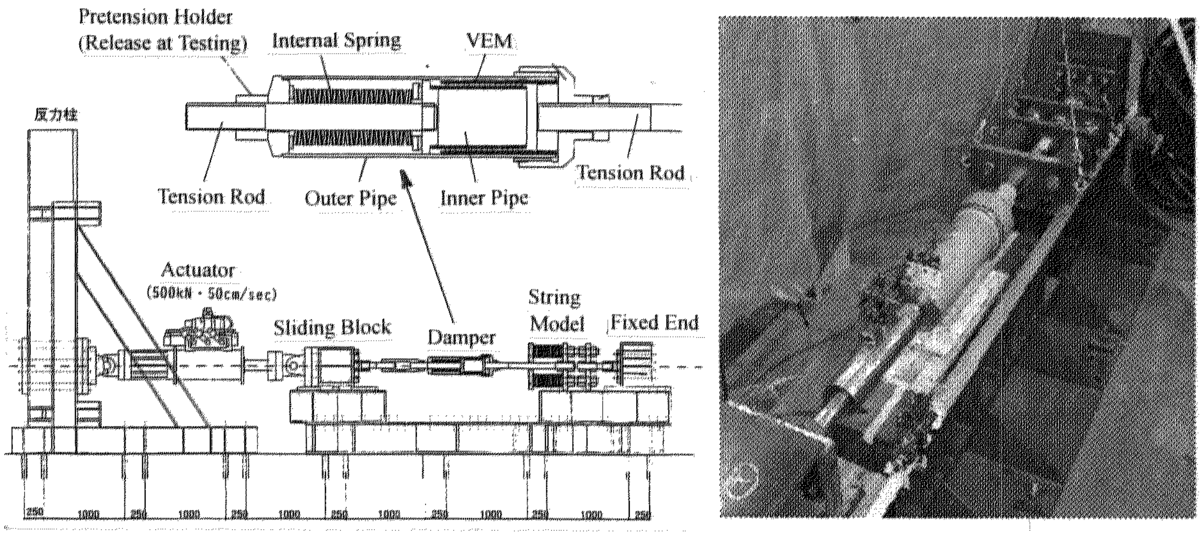



Fig.7. Specimen of Damper and Test Set-up

mm, 5.47N/mm, modeling 36mm-dia.-strings in length of 0.3m, 17m, 38m, respectively. For strings of 17m model and 38m model, mechanical springs with the same stiffness are connected instead of real-length strings, to settle inside the test set-up. With these parameters, specimens of damping mechanisms connected to tension strings of $K_s/K_d$=0.29~5.2, $K_b/K_d$=1.20~∞are composed. Cyclic loading are added on these sets by frequencies of 1.0, 2.0, 3.0Hz, amplitudes of 10, 15,20mm, under temperature of 12℃ and 32℃, with the measurement of their reaction forces and deformation in each parts.

As the results of the test, parts of the load-displacement relationships are shown in Figs. 8. Figure 8a) is the results of $A_d$=1038cm$^2$, $K_s$=1.32kN, $K_b$= 5.47N/mm, 1.0Hz, 10mm, 12℃, and small loops are those of dampers (VEM + internal springs) and large loops are those of dampers with tension strings. Dotted lines in each loops are the calculated theoretical loops by eq.(3)to(12), well expecting the results of the experiments. In case the amplitude grows larger in same conditions the whole system lose reaction forces as shown in Fig.8b), where minimum axial forces go into compression. These actions well model the mechanism of real tension strings with pre-tension forces.

Equivalent stiffness and damping ratio of whole system are calculated from these load-deformation loops, and shown in Figs.9. Figure 9a) is the equivalent stiffness against temperature, and Fig.9b)is the damping ratio against temperature from test results shown as dotted marks, compared with proposed theoretical values shown as lines. In 9b), effect of temperature on damping becomes almost flat, as expected in theoretical studies. Comparison between experimental results and proposed theoretical values for all test cases are summarized in Fig.10. They meets generally within ±15% accuracy, which is the same range for the model of VEM itself, reported in reference 3).

## 5. CONCLUSIONS

1) V(E)M with elastic springs as additional vibration-control dampers are proposed for tension structures. The additional damping, hysteresis loop, and natural period, with respect to the entire structure

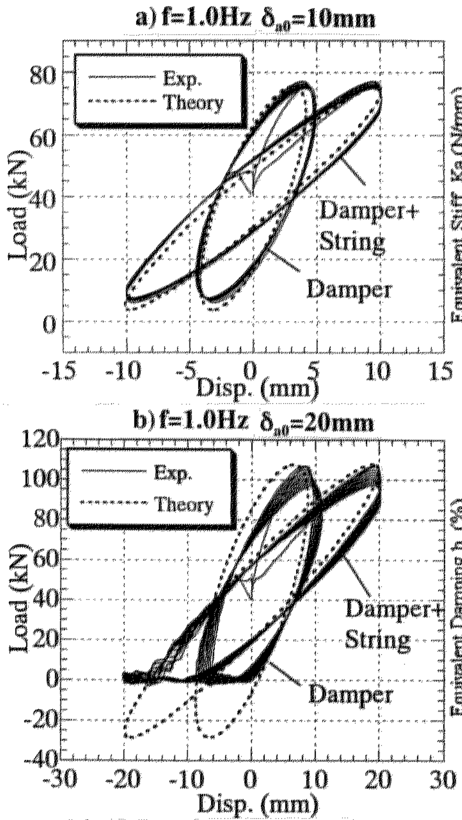



Fig.8 Load-Deflection loops

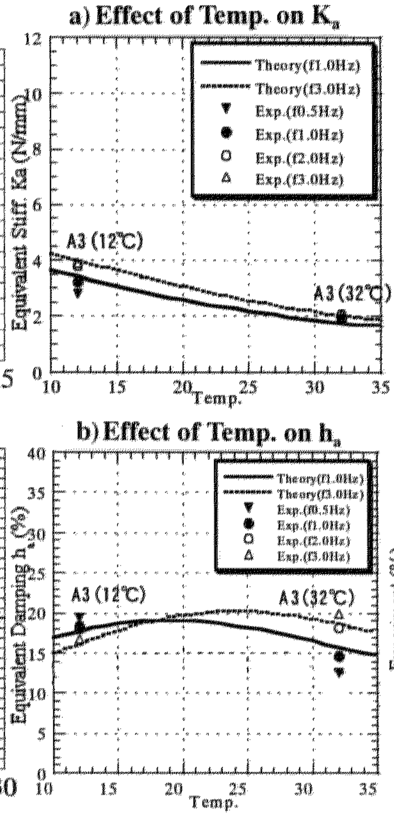



Fig.9 Effect of Tempareture

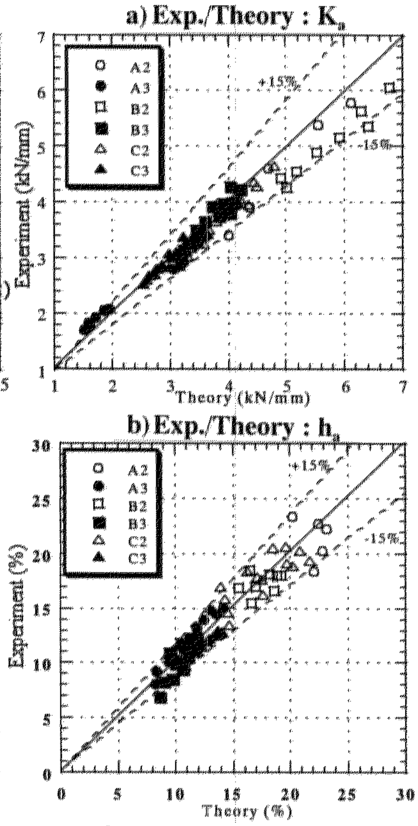



Fig.10 Exp. vs.Theory

can be expressed in an explicit form by taking the internal spring stiffness ratio, connected string stiffness ratio, and frame stiffness ratio with respect to the V(E)M as parameters. The structural characteristics obtained by this method agree well with the results of analysis by a fractional derivative model and real size mock-up tests.

2) The equivalent damping of the entire structure increases with decreasing internal spring stiffness ratio, increasing connected string stiffness ratio, and decreasing frame stiffness ratio. For VEM, when the connected string stiffness ratio $K_b/K_{d0}$ drops to about 4 or less, the temperature dependence of materials appears to decline, due to the connected string to adjust the amplitude of displacement.

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**note:** This Paper composed of additional descriptions with the parts of the following papers:

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