Proceedings of the 8th U.S. National Conference on Earthquake Engineering
April 18-22, 2006, San Francisco, California, USA
Paper No. 318

DYNAMIC INSTABILITY BY LINK-ELEMENT DEFORMATION IN FRAMED-TUBE STRUCTURES

H. Tagawa[1], L. Lowes[2], G. MacRae[3], and A. Wada [4]

ABSTRACT

The influence of link-element deformations on the stability of 3D framed-tube structures during earthquake excitation are evaluated through inelastic dynamic analyses. To minimize computational demand, the 3D framed-tube structures are modeled using 1D MDOF coupled shear-flexural-beam models with elastic link-elements. Dynamic stability of the structure is evaluated using stability coefficients that represent the individual and combined effects of geometric and material nonlinearity. It is found that, as the link-element stiffness decreases, the shear-beam (in-plane frame) and flexural-beam (out-of-plane frame) elements exhibit increasingly different response, which results in larger values of the P-Δ stability coefficient (i.e. larger P-Δ effects). If the link-element between the in-plane and out-of-plane frames has a relatively small stiffness, dynamic structural instability, and potentially collapse, likely will results under earthquake loading.

Introduction

In recent years, structural-control and base-isolation technologies have been used to reduce the seismic forces that are used to design the lateral load resisting frames in super high-rise structures. In these structures, the stiffness of the interior, gravity-resisting framing system is often reduced, sometimes to the point of completely removing the interior framing system. The result is the framed-tube structure (Figure 1), which has perimeter frames comprising deep beams and many small columns. This system has become one of the more popular systems for super high-rise structural systems including the World Trade Center Building (FEMA 403, 2002).

Floor slabs (and gravity-resisting beams, if present) connect out-of-plane frames to in-plane frames as shown in Figure 2. During one-directional shaking the in-plane columns may yield at the top and bottom of each story while the columns in the out-of-plane frames are more likely to remain elastic because they are not subject to direct forces in this direction. Past research (e.g. Gupta and Krawinkler, 2000, Tagawa 2005) indicates that if the link-elements (floor slabs, gravity-resisting beams, and their connections to the out-of-plane walls) have large stiffness and strength, sufficiently large that out-of-plane and in-plane frames have similar displacements, continuous columns in out-of-plane frames are likely to increase structural

[1]COE Researcher, Center for Urban Earthquake Engineering, Tokyo Institute of Technology, Tokyo, Japan.
[2]Assitant Professor, Civil Engineering, University of Washington, Seattle, WA,USA.
[3]Associate Professor, Civil Engineering, University of Canterbury, Christchurch, NZ.
[4]Professor, Structural Engineering Research Center, Tokyo Institute of Technology, Tokyo, Japan.

stability and enhance response. However, if the link-elements are not sufficiently stiff or strong, column buckling or simply significant out-of-plane column deformation may result in unstable structural response, as shown in Figure 3.

This study investigates the impacts of the link-element stiffness on the stability of framed-tube structures. To reduce computational effort, 1D MDOF shear-flexural-beam models with elastic link-elements are used for inelastic dynamic time-history analyses.

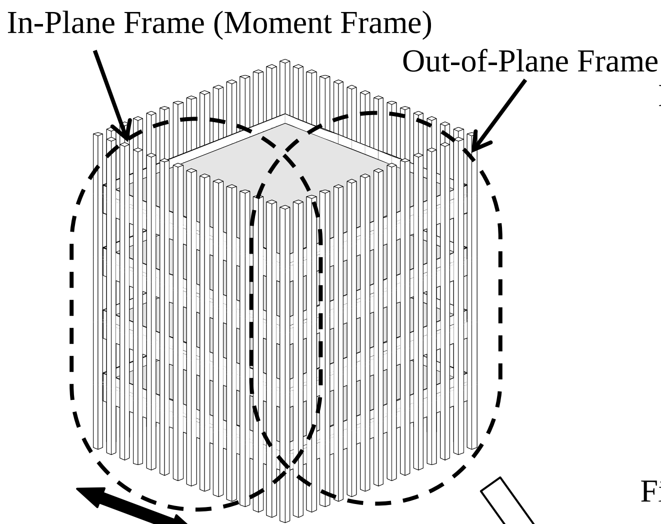


Figure 1: Framed-tube Structure

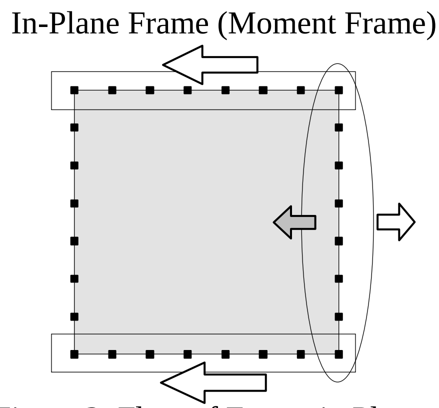


Figure 2: Flow of Forces in Plan

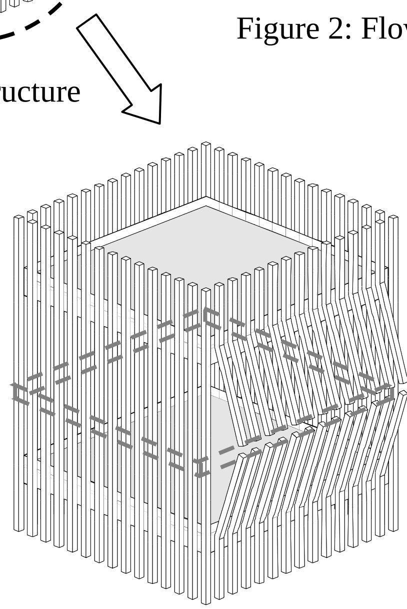


Figure 3: Out-of-Plane Unstable Behavior

Material, Geometric (*P*-Δ), and Net Stability Coefficients

To investigate structural stability and determine the factors that control stability, Tagawa (2005) proposed material, geometric (*P*-Δ), and net stability coefficients:

$$r_{MDOF,i}(t) = \frac{\{\Phi_i(t)\}^T [K(t)] \{\Phi_i(t)\}}{\{\Phi_{o,i}\}^T [K_o] \{\Phi_{o,i}\}} \tag{1}$$

$$\theta_{MDOF,i}(t) = -\frac{\{\Phi_i(t)\}^T [K_g] \{\Phi_i(t)\}}{\{\Phi_{o,i}\}^T [K_o] \{\Phi_{o,i}\}} \tag{2}$$

$$r_{net,i}(t) = r_{MDOF,i}(t) - \theta_{MDOF,i}(t) \tag{3}$$

where $[K(t)]$ is the tangent stiffness matrix at time t, which is assumed to remain constant during the time increment dt, $[K_g]$ is the geometric stiffness matrix, and $\{\Phi_i(t)\}$ is the instantaneous i^{th}-mode shape.The same modal contribution factors are used to normalize the mode-shape vectors, where $[K_o]$ is the initial stiffness matrix. The i^{th}-mode instantaneous eigenvalue of the mass and stiffness matrices, $\Omega_i(t)$, which is a square of the instantaneous frequency, $\omega_i(t)$, is related to the stability coefficients:

$$\Omega_i(t) = r_{net,i}(t) \cdot \frac{\{\Phi_{o,i}\}^T [K_o] \{\Phi_{o,i}\}}{\{\Phi_i(t)\}^T [M] \{\Phi_i(t)\}} = \omega_i(t)^2 \tag{4}$$

where $[M]$ is the mass matrix. When the instantaneous eigenvalue of the structure becomes negative during an earthquake, which means that the instantaneous frequency becomes imaginary, the structure does not "oscillate" and tends to sway in one direction (Bernal 1998, Uetani and Tagawa, 1998, Araki and Hjelmstad, 2000). As a result, large maximum and residual story drifts tend to result.

1D MDOF Coupled Shear-Flexure-Beam Model with Link

To reduce computational demands and perform stability analysis at each-time step, the 3D framed-tube structure is modeled using the 1D MDOF shear-flexural-beam model with an elastic link-element. The moment-resisting frame in the direction of horizontal shaking is represented by the shear-beam and the out-of-plane frame is represented by the flexural-beam, as shown in Figure 2. Floor slabs and their connections which are in the load path between the in-plane and out-of-plane frames are modeled by link-elements in the 1D MDOF model. The incremental form of the equation of motion of this structural system is given by Equation 5, where $[M]$ is the mass matrix, $[C]$ is the viscous damping matrix, $[K]$ is the instantaneous stiffness matrix, $\{\Delta_s\}$ is a vector of relative displacements of the shear-beam, $\{\Delta_f\}$ is a vector of the relative displacement of the flexural-beam, and Δ_g is a ground motion acceleration. As shown in Equation 6, the stiffness matrix, $[K]$, is the sum of the stiffness matrices of shear-beam, $[K_{shear}]$, given by Equation 7, flexural-beam, $[K_{flexure}]$, given by Equation 11, and link element, $[K_{link}]$, given by Equation 12. Since the flexural-beam stiffness matrix, [$K_{flexure}$], needs to be expressed in terms of rotational degree-of-freedom, static condensation is carried out using the boundary conditions, [M_{total}] = 0. In this study, the link-element is an elastic spring.

$$[M]\cdot d\begin{Bmatrix}\ddot{\Delta}_s\\ \ddot{\Delta}_f\end{Bmatrix}+[C]\cdot d\begin{Bmatrix}\dot{\Delta}_s\\ \dot{\Delta}_f\end{Bmatrix}+[K]\cdot d\begin{Bmatrix}\Delta_s\\ \Delta_f\end{Bmatrix}=-[M][I]d\ddot{\Delta}_g \tag{5}$$

$$[K]=\begin{bmatrix}[K_{shear}]+[K_{link}] & -[K_{link}]\\ -[K_{link}] & [K_{flexure}]+[K_{link}]\end{bmatrix}_{2n\times 2n} \tag{6}$$

<u>(1) Shear-beam Stiffness Matrix, [K_{shear}]</u>

$$[K_{shear}]=\begin{bmatrix}k_1^s+k_2^s & -k_2^s & 0 & \cdots & 0\\ & k_2^s+k_3^s & 0 & \ddots & \vdots\\ & & \ddots & 0 & 0\\ & sym. & & k_{n-1}^s+k_n^s & -k_n^s\\ & & & & k_n^s\end{bmatrix} \tag{7}$$

$$k_i^s=\frac{12E\left(\sum I_i^{SC}\right)}{H_i^3} \tag{8}$$

<u>(2) Flexural-beam Stiffness Matrix, [$K_{flexure}$]</u>

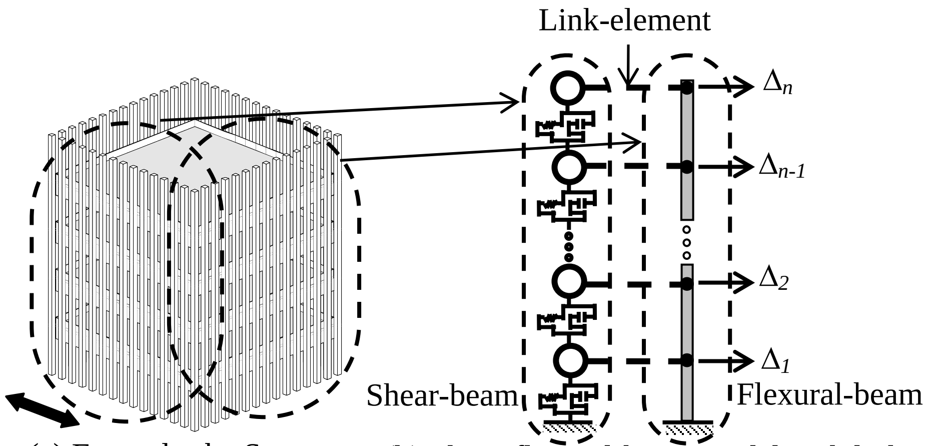
$$\begin{Bmatrix}F_i^a\\ M_i^a\\ F_i^b\\ M_i^b\end{Bmatrix}=\frac{E\left(\sum I_i^{fc}\right)}{H_i^3}\begin{bmatrix}12 & 6H_i & -12 & 6H_i\\ & 4H_i^2 & -6H_i & 2H_i^2\\ & & 12 & -6H_i\\ sym. & & & 4H_i^2\end{bmatrix}\begin{Bmatrix}\Delta_i^a\\ \Delta_i^a\\ \Delta_i^b\\ \Delta_i^b\end{Bmatrix} \tag{9}$$

$$\begin{Bmatrix}\{F_{total}\}\\ \{M_{total}\}\end{Bmatrix}=\begin{bmatrix}[K_{\Delta\Delta}] & [K_{\Delta\theta}]\\ [K_{\theta\Delta}] & [K_{\theta\theta}]\end{bmatrix}\begin{Bmatrix}\{\Delta_{total}\}\\ \{\theta_{total}\}\end{Bmatrix} \tag{10}$$

$$[K_{flexure}]_{n\times n}=\frac{\{F_{total}\}}{\{\Delta_{total}\}}=\left[K_{\Delta\Delta}\right]-\left[K_{\Delta\theta}\right]\left[K_{\theta\theta}\right]^{-1}\left[K_{\theta\Delta}\right] \tag{11}$$

<u>(3) Link-element Stiffness Matrix, [K_{link}]</u>

$$\left[K_{link}\right]=\begin{bmatrix}k_1^l & 0 & \cdots & \cdots & 0\\ & k_2^l & \ddots & \ddots & \vdots\\ & & & \ddots & \vdots\\ & sym. & & k_{n-1}^l & 0\\ & & & & k_n^l\end{bmatrix} \tag{12}$$



(a) Framed-tube Structure (b) Shear-flexural-beam model with link

Figure 2: Configuration of 1D coupled shear-flexural-beam model with link.

Structural Properties

A series of 1D MDOF coupled shear-flexural-beam models, with elastic link-elements, were created of a 10-story framed-tube structure. The following assumptions regarding original structures, analyzed models, and analysis method are made:

- The 1st-story height was taken as 216in (5.49m). Other stories were assumed to have a height of 156in (3.96m).
- Assuming the geometry of in-plane and out-of-plane frames to be identical, the flexural-beam stiffness was determined so that $\kappa_{c.c.}$ =1.0, where $\kappa_{c.c.}$ is the ratio of the flexural-beam stiffness to the shear-beam stiffness.
- The flexural-beam was assumed to be elastic during an earthquake. The flexural-beam was fixed to the ground at the base.
- It is assumed that the weight is distributed evenly in the shear-beam and the flexural-beam.
- The link-element was assumed to have a stiffness, K_{link}, that is uniform over the height of the structure. Multiple versions of the model were created by varying link-element stiffnesses from zero to infinity.
- The initial stiffness of the shear-beam was determined so that story drift angles would be identical over the height of the building under the lateral force distribution defined by IBC (2003). With this stiffness, the natural period of the structures was 2.5s assuming that the mass of the framed-tube structure is centered at the shear-beam nodes, that all stories have the same mass, and that the link-elements are rigid.
- The story force reduction factor, R_s, of the shear-beam was set at 4.
- The post-yield tangent stiffness ratio for each story in the shear-beam was assumed to be 3%.
- A Rayleigh damping ratio of 2% at the periods of 2.5s and 0.2s was used.
- The geometric matrices for each element of the shear-beam and the flexural-beam were derived from linear and cubic functions, respectively.

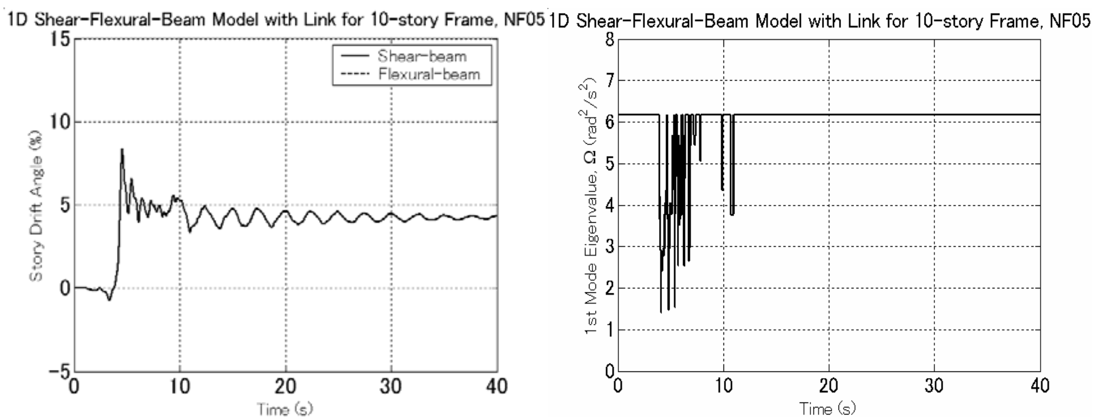
The earthquake record used is the NF05 in the 40 SAC NF records (Summerville, et al., 1997).

Impacts of Link-Element Deformation on Frame Stability

The impact of link-element stiffness on structural stability was investigated by simulating the dynamic response of the framed-tube structure with link-element stiffnesses that varied from zero to infinity. The following sections provide the results of these analyses.

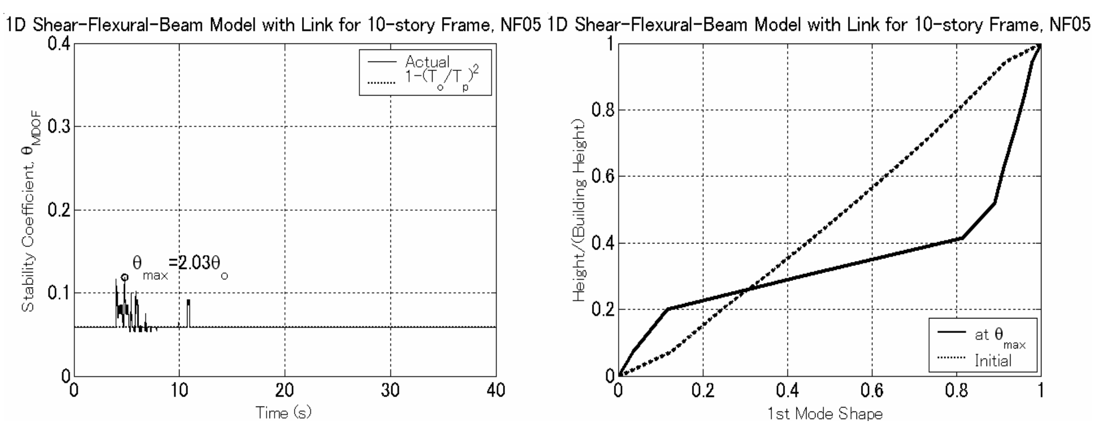
(1) $K_{link} = \infty$

First, K_{link} was defined to be approximately rigid. The results of these analyses are shown in Figure 3 and Figure 4. For this value of K_{link}, there is no runaway drift in the story with the highest story drift angle (SDA) as shown in Figure 3(a). This is consistent with the fact that the eigenvalue remains positive even though the P-Δ stability coefficient (Eq. 2) increases to 203% of the initial elastic value as shown in Figure 4(a). The increase of the P-Δ stability coefficient is due to drift concentrations that develop in the instantaneous 1st-mode shape as shown in Figure 4(b). Here, since the link-element stiffness is very large, the mode shapes of the shear-beam and flexural-beam are identical.



(a) SDA history in the story with maximum SDA (b) Instantaneous eigenvalue

Figure 3: SDA history and eigenvalues in 1D MDOF coupled model ($\kappa_{c.c}$ =1.0, $K_{link} = \infty$).

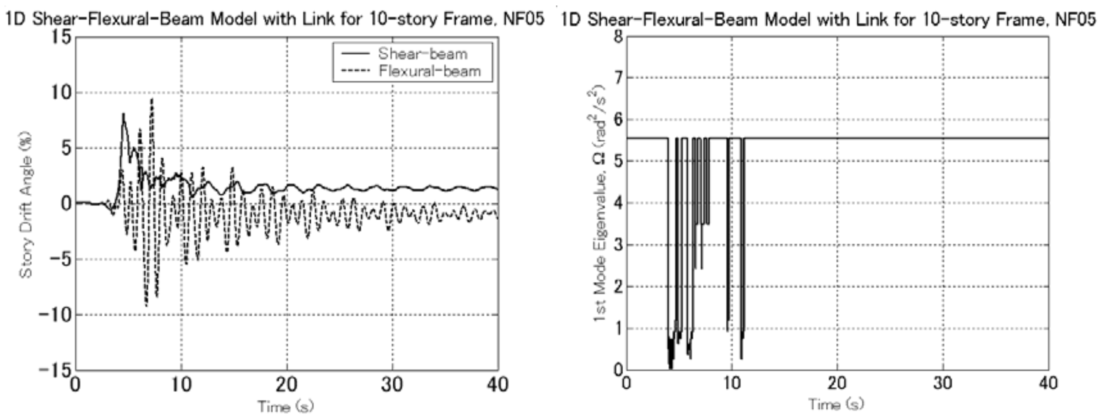


(a) P-Δ stability coefficient (b) Instantaneous 1st-mode shape

Figure 4: θ_{MDOF} and 1st-mode shape at maximum θ_{MDOF} in 1D model ($\kappa_{c.c}$ =1.0, $K_{link} = \infty$).

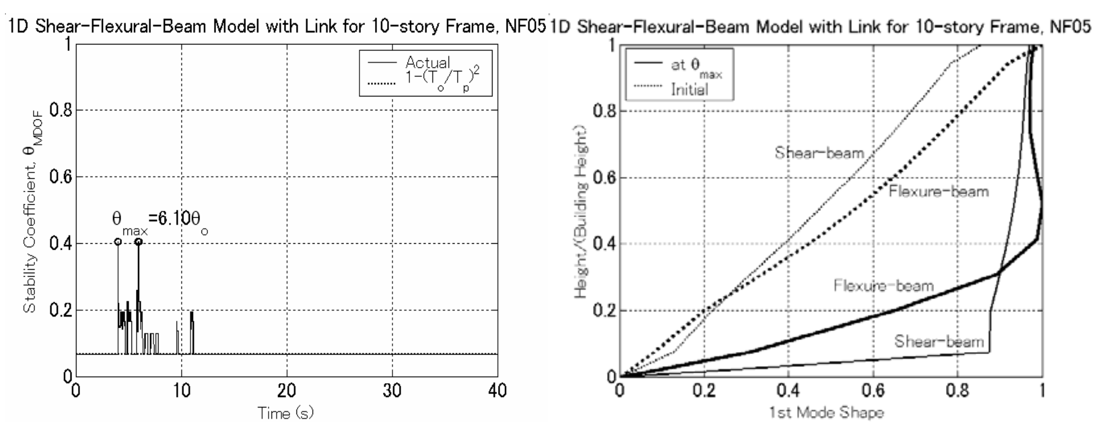
(2) K_{link} = 20 (kips/in)

Next, K_{link} was defined to be 20 kips/in. The results of analyses with this stiffness are shown in Figure 5 and Figure 6. Here, the structure moves in one direction and large maximum and residual interstory drifts result, as shown in Figure 5(a). These large drifts are reflected in the small positive and negative eigenvalues that occur during the ground motion, as shown in Figure 5(b). For this structure, the P-Δ stability coefficient increases to 610% of the initial elastic value, as shown in Figure 6(a). Due to the link-element flexibility, the shear-beam and flexural-beam have different displacements during the earthquake. The mode shapes of the shear-beam and flexural-beam are shown in Figure 6(b). Thus, it can be concluded that the link-element deformation, and the resulting relative displacement of the shear- and flexural-beam components, increase the P-Δ effects.



(a) SDA history in the story with maximum SDA (b) Instantaneous eigenvalue

Figure 5: SDA history and eigenvalues in 1D MDOF coupled model ($\kappa_{c.c}$ =1.0, K_{link} = 20kips/in).



(a) P-Δ stability coefficient (b) Instantaneous 1st-mode shape

Figure 6: θ_{MDOF} and instantaneous 1st-mode shape at maximum θ_{MDOF} in 1D MDOF coupled model ($\kappa_{c.c}$ =1.0, K_{link} = 20kips/in).

(3) K_{link} = 0

For the final analyses, K_{link} was defined to be zero. The maximum SDA time-history for the structure is shown in Figure 7. For this structure, the shear- and flexural-beam behave independently. The flexural-beam moves in one direction and collapses. The shear-beam moves to one-direction and exhibits large maximum and residual drifts.

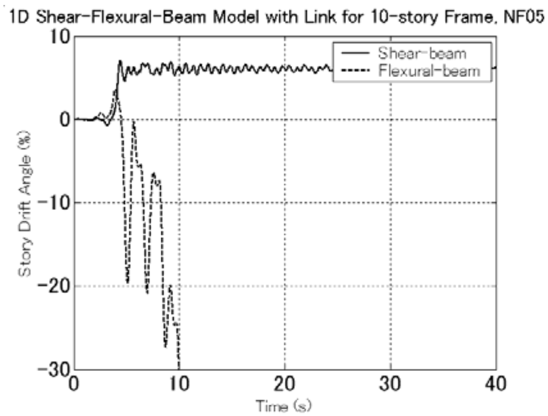
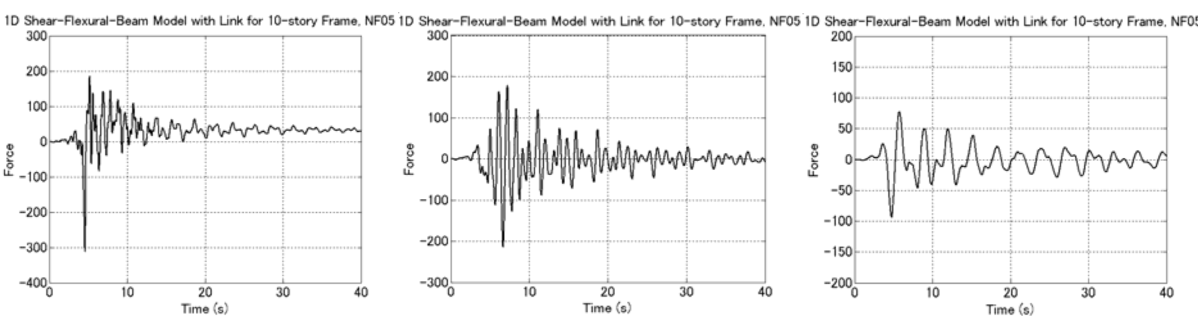


Figure 7: SDA history and eigenvalues in 1D MDOF coupled model ($\kappa_{c.c}$ =1.0, K_{link} = 0).

Required Link-Element Strength

The link-element load, which would be the strength required to ensure that the link element remains elastic during the earthquake, is shown for three link-element stiffnesses, K_{link} = ∞, 20 kips/in, and 2 kips/in. The mass is assumed to be distributed evenly to the shear-beam and flexural-beam. This may not be realistic since more than one half of the mass may be attributed to the shear-beam if the link-element between the out-of-plane frame and the floor is more flexible than that between the in-plane frame and the floor. The time-history of force acting on the 6th-story link-element is shown in Figure 8. It is found that as the link-element stiffness decreases, the required link-element strength, for the link element to remain elastic, decreases and the vibration period of the link element increases.

In this study, the link-element is assumed to be elastic. In realistic structures, however, the frame bears onto the floor slab and the link-element is expected to be stiff and strong in compression. On the other hand, past documents (e.g. FEMA 403, 2002) indicates that, in tension, the link-element has some flexibility and small strength, for example, with an estimated strength of about 90kips in the World Trade Center Building (FEMA 403, 2002). This implies that the link-element would fail in tension since the maximum tension force acting on the link-element in the 1D model with K_{link} = ∞ is approximately 180kips, which is greater than the capacity of approximately 90kips, as shown in Figure 8(a). More realistic hysteretic loops of the link-element, which are different in tension and compression, should be modeled in further research.

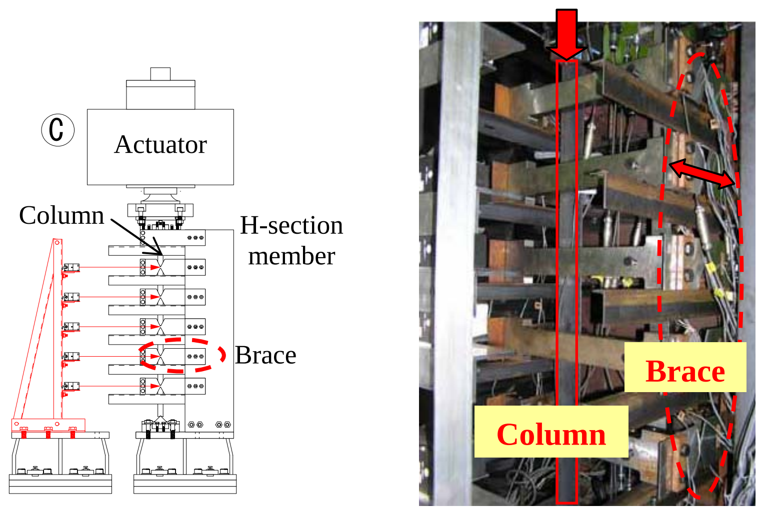


(a) K_{link} = ∞ (b) K_{link} = 20 kips/in (c) K_{link} = 2 kips/in

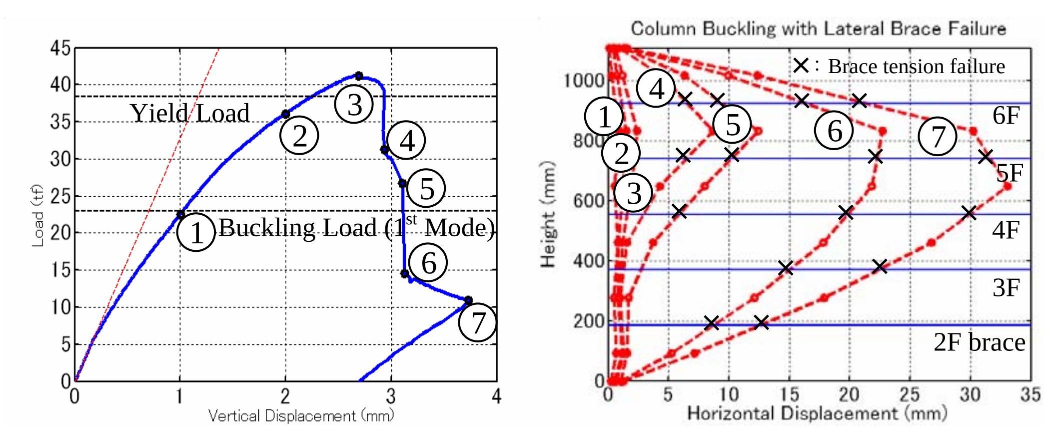
Figure 8: Link-element force histories for 1D MDOF coupled model with various K_{link}, NF05.

Ongoing Experiments

Currently, a number of experiments on the column buckling with the lateral bracing, which intends to simulate the unstable behavior of the framed-tube structures, are being carried out at the Structural Engineering laboratory in the Tokyo Institute of Technology. Figure 9 shows the specimen consisting of a steel column (slenderness parameter, L/r = 120) and 5 lateral braces made of aluminum for a simplified modeling of a 6-story building. The column simulates the out-of-plane column in the frame-tube structure. The aluminum brace intends to provide small stiffness and strength in tension when the column deforms outward. When the column deforms inward, the movement is constrained by H-section steel member. This intends to simulate the compressive resistance of the floor system in the framed-tube structure. Figure 10(a) shows the vertical load (*P*) - vertical displacement (*v*) relationship and Figure 10(b) shows the horizontal deformation of the column along the height at the stages of (1) v = 1mm, (2) v = 2mm, (3) maximum load, (4) the break of the 5th floor (5F) and 6F braces, (5) the break of the 4F brace, (6) the break of the 3F and 2F braces, and (7) unloading. It is observed that the deflection mode shape changes dramatically with the sudden decrease of the load after the progressive notch failure in tension.



(a) Elevation of specimen and test device (b) Picture of specimen and test device
Figure 9: Experiments on column buckling with rupture of lateral bracing.



(a) Load – vertical displacement (b) Mode shape of horizontal displacement
Figure 10: Results for column buckling with rupture of lateral brace.

Conclusions

The impact of continuous column stiffness and link-element deformation on the dynamic stability and response of the 3D framed-tube structure was investigated. A 1D coupled shear-flexural-beam model with elastic link-elements was used to model the structural system. The P-Δ, material, and net stability coefficients proposed previously by Tagawa (2005) were used to evaluate structural stability. Major findings of the study are:

1) If the link-element between the shear-beam and flexural-beam components of the structure is perfectly rigid, the flexural-beam provides additional stiffness to the structure when the shear-beam yields and has a negative tangent stiffness due to P-Δ effects.
2) If the link-element between the shear-beam and flexural-beam deforms, the P-Δ stability coefficient increases and the structural instability is more likely.
3) As the link-element stiffness decreases, the strength required for the link-element, to ensure that the link-element remains elastic under earthquake loading, becomes smaller.

Acknowledgement

The authors would like to express sincere appreciation to Prof. Satoshi Yamada in the Tokyo Institute of Technology for his kind advices on the experiments, and to Mr. Shinya Sumitani, a graduate student in the Tokyo Institute of Technology, for carrying out the experiments and creating several figures in this paper.

References

Araki, Y. and Hjelmstad K., 2000. *Criteria for Assessing Dynamic Collapse of Elastoplastic Structural Systems*, Earthquake Engineering and Structural Dynamics, Vol. 29.

Bernal, D. 1998., *Instability of buildings during seismic response*, Engineering Structures, 20(4-6).

FEMA-403, 2002. *World Trade Center Building Performance Study: Data Collection, Preliminary Observations, and Recommendations*, FEMA, Washington, DC.

Gupta, A. and Krawinkler, H., 2000. *Dynamic P-Delta Effects for Flexible Inelastic Steel Structures*, Journal of Structural Engineering, ASCE 126(1): 145-154.

Jennings, P. and Husid, R., 1968, Collapse of Yielding Structures under Earthquakes, Journal of Engineering Mechanics Division, ASCE, 94.

Somerville, P., Smith, N., Punyamurthula, S., and Sun, J., 1997. *Development of ground motion time histories for phase 2 of the FEMA/SAC steel project*, Rep. No. SAC/BD97/04, SAC Joint Venture, Sacramento, California.

Tagawa, H., 2005. *Towards an Understanding of Seismic Response of 3D Structures – Stability & Reliability,* Partial Requirement for Doctorate Thesis, University of Washington, Seattle.

Uetani, K. and Tagawa, H., 1998, *Criteria for Suppression of Deformation Concentration of Building Frames under Severe Earthquakes*, Engineering Structures, Vol. 20.